



# Cambridge International AS & A Level

CANDIDATE  
NAMECENTRE  
NUMBER

|  |  |  |  |  |
|--|--|--|--|--|
|  |  |  |  |  |
|--|--|--|--|--|

CANDIDATE  
NUMBER

|  |  |  |  |
|--|--|--|--|
|  |  |  |  |
|--|--|--|--|

## FURTHER MATHEMATICS

9231/14

Paper 1 Further Pure Mathematics 1

May/June 2026

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

### INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen. Do **not** use correction fluid or tape.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.



- 1 (a)** Use standard results from the list of formulae (MF19) to show that

$$\sum_{r=1}^n (r-1)^2 = an^3 + bn^2 + cn,$$

where  $a$ ,  $b$  and  $c$  are constants to be determined.

[2]





It is given that  $u_r = \frac{1}{r^2 - 2r + 1} - \frac{1}{r^2 + 2r + 1}$ .

- (b) Use the method of differences to find  $\sum_{r=2}^n u_r$  in terms of  $n$ . [4]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (c) State the value of  $\sum_{r=2}^{\infty} u_r$ . [1]

.....

.....

.....

.....

.....





**2** The equation  $2x^4 + 3x^3 + x + 1 = 0$  has roots  $\alpha, \beta, \gamma, \delta$ .

(a) Find a quartic equation whose roots are  $\frac{1}{\alpha^3}, \frac{1}{\beta^3}, \frac{1}{\gamma^3}, \frac{1}{\delta^3}$  and state the value of  $\frac{1}{\alpha^3} + \frac{1}{\beta^3} + \frac{1}{\gamma^3} + \frac{1}{\delta^3}$ .

[5]

(b) Find the value of  $\frac{1}{\alpha^6} + \frac{1}{\beta^6} + \frac{1}{\gamma^6} + \frac{1}{\delta^6}$ .

[2]



- 3 The sequence of positive numbers  $u_1, u_2, u_3, \dots$  is such that  $u_1 > 1$  and, for  $n \geq 1$ ,

$$u_{n+1} = \frac{u_n^2 + 2u_n + 2}{u_n + 4}.$$

- (a) By considering  $u_{n+1} - 1$ , or otherwise, prove by mathematical induction that  $u_n > 1$  for all positive integers  $n$ . [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Show that  $u_{n+1} < u_n$  for  $n \geq 1$ . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



4 The matrix  $\mathbf{A}$  is given by

$$\mathbf{A} = \begin{pmatrix} 1 & k & -1 \\ 3 & 1 & k \\ 3 & 1 & -1 \end{pmatrix},$$

where  $k$  is a real constant.

(a) Find the values of  $k$  for which  $\mathbf{A}$  is singular. [3]

.....

.....

.....

.....

.....

.....

.....

.....

The matrices  $\mathbf{B}$  and  $\mathbf{C}$  are given by

$$\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \\ 1 & 0 \end{pmatrix} \text{ and } \mathbf{C} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}.$$

(b) Show that  $\mathbf{CAB} = \begin{pmatrix} k+3 & -1 \\ 2 & -k-1 \end{pmatrix}$ . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....





Show that the transformation in the  $x$ - $y$  plane represented by **CAB** has no invariant lines. [5]

[5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

5 The points  $A, B, C$  have position vectors

$$4\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}, \quad 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}, \quad 5\mathbf{i} + 2\mathbf{j} + 2\mathbf{k},$$

respectively, relative to the origin  $O$ .

(a) Find the equation of the plane  $ABC$ , giving your answer in the form  $ax + by + cz = d$ . [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

The point  $D$  has position vector  $2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ .

(b) Find the acute angle between the line  $OD$  and the plane  $ABC$ . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....





(c) Find the shortest distance between the lines  $AB$  and  $CD$ .

[5]

[illegible]



- 6 The curve  $C$  has equation  $y = f(x)$ , where  $f(x) = \frac{2x^2 - x + 1}{2x^2 - 2x}$ .

(a) Find the equations of the asymptotes of  $C$ .

[2]

.....

.....

.....

.....

.....

.....

.....

- (b) Find the coordinates of any stationary points on  $C$ , giving your answers correct to 1 decimal place.

[4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



(c) Sketch  $C$ .

[3]

(d) Sketch the curve with equation  $y = \frac{1}{f(x)}$  and state the set of values for which  $\frac{1}{f(x)} < f(x)$ . [4]

- 7 (a) Show that the curve with Cartesian equation

$$(x^2 + y^2)^{\frac{3}{2}} = xy$$

has polar equation  $r = \frac{1}{2} \sin 2\theta$ . [3]

.....

.....

.....

.....

.....

.....

The curves  $C_1$  and  $C_2$  have polar equations

$$r = \frac{1}{2} \sin 2\theta \quad \text{and} \quad r^2 = \frac{1}{\left(\theta - \frac{1}{4}\pi\right)^2 + 5}$$

respectively, where  $0 \leq \theta \leq \frac{1}{2}\pi$ .

- (b) Show that  $C_1$  and  $C_2$  intersect where

$$\left(\left(\theta - \frac{1}{4}\pi\right)^2 + 5\right) \sin^2 2\theta = 4$$

and verify by calculation that this equation has a root  $\alpha$  between  $\theta = 0.5$  and  $\theta = 0.6$ . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....





- (c) Sketch  $C_1$  and  $C_2$  on the same diagram, clearly identifying each curve and any lines of symmetry.

[4]

- (d)** The region  $R$  is enclosed by  $C_1$  and  $C_2$  and includes the pole. Show that the area of  $R$  is given by

$$\frac{1}{32}(4\alpha - \sin 4\alpha) + \frac{1}{5}\sqrt{5} \tan^{-1}\left(\frac{1}{5}\sqrt{5}\left(\frac{1}{4}\pi - \alpha\right)\right),$$

where  $\alpha$  is the root found in 7(b).

[6]

[illegible]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

## Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (Cambridge University Press & Assessment) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in our Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at [www.cambridgeinternational.org](http://www.cambridgeinternational.org) after the live examination series.

Cambridge International Education is the name of our awarding body and a part of Cambridge University Press & Assessment, which is a department of the University of Cambridge.

